MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

DIM5068 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

19 OCTOBER 2019
2.30 pm - 4.30 pm
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

a. Given the function of $y=\sin\left(3x^6+7\ln x\right)$. Find the first derivative by using the **Chain Rule.** (3.5 marks)

b. Given the function of $z=4x^{-2}+3e^{2x}-5\cos x$. Find $\frac{d^3z}{dx^3}$ and express your answer in **positive exponent.** (4.5 marks)

c. For the function $f(x)=\frac{x^3}{3}+x^2-15x$,

i) Find the **critical number(s).** (2 marks)
ii) Identify the intervals on which f is increasing or decreasing. (3 marks)
iii) Determine the maximum and/or the minimum value(s). (2 marks)

[TOTAL 15 MARKS]

Question 2

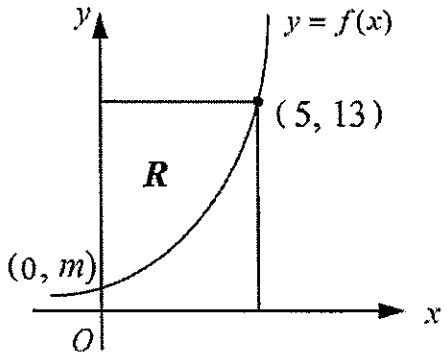
a. Evaluate $\int 6x-\frac{5}{x}+3\,dx$. (2 marks)

b. Show that the value of $a=-2$ and $b=4$ satisfy the equation of $\int_a^b\left(3x^2+\frac{1}{2}x\right)dx=75$. (2.5 marks)

c. Given $\int_1^3 p(x)dx=107$ and $\int_3^6 p(x)dx=118$. Find $\int_1^6 p(x)-3x^2\,dx$ (4.5 marks)

d. Use **Integration by Parts** to solve $\int 2xe^x\,dx$. (4.5 marks)

e. Diagram below shows a curve $y=f(x)$ which passes through the points (0,m) and (5,13). If $\int_0^5 f(x)dx=15$, find the area of the region ***R***.



(1.5 marks)

[TOTAL 15 MARKS]

Continued...

Question 3

a. Given the first order differential equation $\frac{dy}{dx} = ye^x + 2y$,

i) Solve by using the **separable method**. (2 marks)

ii) From your answer in part a(i), determine the solution of the initial value problem if $y(0)=1$. (1.5 marks)

b. Given the differential equation $x^2\frac{dy}{dx} + xy = x\sin x + 3x^3$,

i) Find $p(x)$ and $q(x)$. (1.5 mark)

ii) Solve for y by using the **method of integrating factors**. (3 marks)

c. Find the **general solution** of homogeneous equation $16y'' - 8y' + y = 0$. (2 marks)

[TOTAL 10 MARKS]

Question 4

a. Given that vectors $\mathbf{m} = \langle 2,-2,7\rangle$ and $\mathbf{n} = \langle 3,10,2\rangle$.

i) Find the dot product of **m** and **n**. (1 mark)

ii) Based on the result in part (i), determine whether **m** and **n** are orthogonal. Why? (1 mark)

b. Sany wants to determine the area of the triangle with vertices ***P***, ***Q*** and ***R***. Given that vector $\vec{PQ} = \langle 2,1,-5\rangle$ and vector $\vec{PR} = \langle 1,2,-1\rangle$,

i) Calculate the **cross product** of $\vec{PQ}$ and $\vec{PR}$. (3 marks)

ii) Compute the total area of the triangle. (2 marks)

c. If a line passes through a point $(3,-5,1)$ and is parallel to a vector $\vec{v} = 2\vec{i} + 7\vec{j} - 4\vec{k}$, compute the **vector equation** and **parametric equation** for the line. (3 marks)

[TOTAL 10 MARKS]

End of page.

APPENDIX

Derivatives: $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Differentiation Rules

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}\left(x^n\right)=nx^{n-1}$
4. $\dfrac{d}{dx}[f(u)]=\dfrac{dy}{du}.\dfrac{du}{dx}$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}\left(e^x\right)=e^x$
2. $\dfrac{d}{dx}\left(a^x\right)=a^x\ln a$
3. $\dfrac{d}{dx}\left(\ln x\right)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}\left(\log_a x\right)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}\left(\sin x\right)=\cos x$
2. $\dfrac{d}{dx}\left(\cos x\right)=-\sin x$
3. $\dfrac{d}{dx}\left(\tan x\right)=\sec^2 x$
4. $\dfrac{d}{dx}\left(\csc x\right)=-\csc x\cot x$
5. $\dfrac{d}{dx}\left(\sec x\right)=\sec x\tan x$
6. $\dfrac{d}{dx}\left(\cot x\right)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int\sin u\,du=-\cos u+C$
6. $\int\cos u\,du=\sin u+C$
7. $\int\sec^2 u\,du=\tan u+C$
8. $\int\csc^2 u\,du=-\cot u+C$
9. $\int\sec u\tan u\,du=\sec u+C$
10. $\int\csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b[f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx$, where $\mu = e^{\int p(x)\,dx}$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$= \left| \vec{PQ} \times \vec{PR} \right|$

Area for triangle PQR

$= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

Symmetric equation: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} \right)$